MICROWAVE BRIDGES

VANCE HOBERT HUDGINS

Library
U. S. Naval Postgraduate School
Monterey, California

Mont 225
8854

Library
U. S. Naval Postgraduate School
Annapolis, Md.

# MICROWAVE BRIDGES

by

VANCE HOBERT HUDGINS

An Essay

Submitted to The Advisory Board of

The School of Engineering

of

The Johns Hopkins University

in conformity with the requirements for the

Degree of Master of Science in Engineering

Baltimore Maryland

May 1950

Thesis
4847

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. J. V. Lebacqz of The Johns Hopkins University for suggesting this topic and for his assistance and cooperation throughout the course of the investigation. Appreciation is also expressed to Dr. C. F. Miller and to the other members of the faculty and the department who aided considerably in this work.

13248

68981

TABLE OF CONTENTS


1. SUMMARY 1

2. INTRODUCTION 2

3. PROPAGATION 4

4. HYBRID DEVICES 7

5. MAGIC TEE 9

   a. Basic Theory 9

   b. Operating Procedure for Measuring VSWR 17

   c. Design Considerations 18

   d. Relative Merits of the Tee Bridge and the Directional Coupler 19

   e. Use of the Tee Bridge About A Point Other than Match 20

   f. Measurement of Phase 20

6. HYBRID RING OR "RAT-RACES" 22

   a. Design Considerations 30

   b. Frequency Characteristics 30

7. APPLICATIONS 32

8. SIX-ARM [illegible] 38

   a. Design Considerations and Bridge Characteristics 42

   b. Four Terminal Application 45

9. CONCLUSION 46

10. APPENDIX A 47

11. APPENDIX B 54

12. APPENDIX C 65

13. BIBLIOGRAPHY 68

14. VITA 71

## SUMMARY

This paper reviews and discusses the fundamental behavior of several microwave bridges. Some basic theory and design consideration is given. This paper is qualitative in nature with some mathematical analysis given in the appendices. The propagation characteristics in wave-guides and the phase properties of simple wave-guide tee junctions are briefly considered. Two kinds of hybrid circuits are then described, the one involving a ring or loop of transmission lines, the other relying upon the symmetry properties of certain four-arm junctions. The description is centered about wave-guide structures for microwaves, but the principles may also be applied to other kinds of transmission lines for other frequency ranges. The relative merits of directional couplers and tee bridge elements are discussed. A wave-guide structure, which is a true microwave equivalent of a Wheatstone bridge, is discussed in detail.

Some of the more important applications are outlined and a discussion of the merits of each bridge is given.

## INTRODUCTION

The wave-guide has been developed in the last fifteen years into a very important medium for microwave transmission. During the war progress was especially rapid due to the use of wave-guides in radar. Many of the most important developments in radar would have been virtually impossible without the use of these guides. It is now possible to foresee some of the probable uses of wave-guides in communications developments of the near future. Hollow-pipe guides are not practical until the wavelength is shorter than about thirty centimeters. At this limit both coaxial cables and wave-guides are used. As the wavelengths get shorter, the wave-guide becomes increasingly attractive until at ten centimeters it is preferred for efficient, shielded transmission of power, and at three centimeters it is used almost exclusively.

Although the earliest work emphasized the wave-guide as a form of transmission line for radio waves, it was apparent from the outset that wave-guide structures would be required also to serve as numerous basic electrical components. These components at low frequencies include such elements as resistors, transformers, relays, switches, coils, capacitors, etc. Wave-guide counterparts of these low frequency components have been constructed to operate in the microwave region. This paper presents a discussion of one group of these components called "Microwave Bridges."

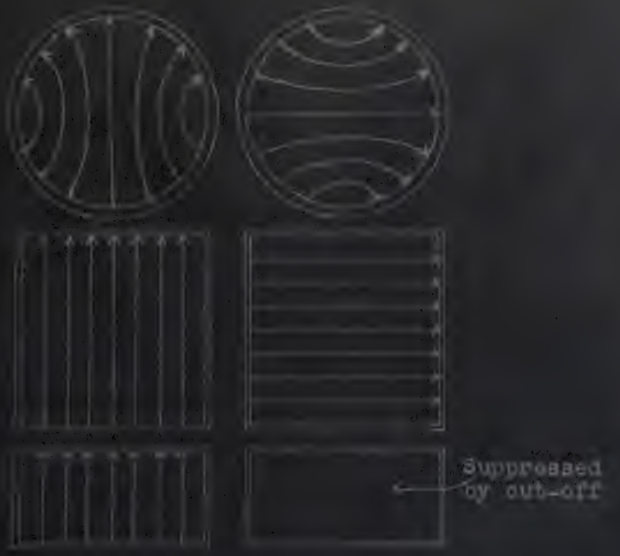
The behavior of microwave bridges is similar to that of the conventional low-frequency bridge circuits which compare the load impedances of the collinear arms, the signal being fed in one branch arm and a detector on the other. A microwave bridge consists of three groups of components: r-f sources, the bridge element, and the detector-indicator. Of these, the bridge element is the most interesting since it must couple to the detector only that power which is reflected from the test piece. Therefore, most of this paper is concerned with a discussion of the bridge elements. The electrical behavior, physical form, possible applications, and other characteristics of several such bridge elements will be discussed.

## PROPAGATION

It is well known that radio waves can be freely propagated within any conducting tube, so long as the wavelength is shorter than a cutoff wavelength determined by the cross sectional size and shape of the pipe. There are many transmission modes, corresponding to different patterns of lines of force, which can be established and maintained for wave transmission, and thus many different cutoff wavelengths. However, for reasons of simplicity, wave-guide developments have so far been confined almost entirely to the mode of longest cutoff wavelength, the so-called dominant wave, in pipes so proportioned that no other mode can be freely sustained. Lines of electric intensity in the dominant wave are shown in Figure 1 for tubes of circular, square, and oblong cross section. There is evidently a close resemblance to radio waves. For practical reasons circular and rectangular shapes are in general use; however, other shapes are being introduced.

Patterns are shown in Figure 1 for both vertical and horizontal polarization. In the circular and square wave guides, the electric field configuration may have any orientation, depending upon the manner in which the waves were initially launched. However, in the rectangular guide the cutoff wavelength corresponding to the cross-polarized oscillation can be depressed below the operating wavelength, and only the fixed orientation shown can be freely transmitted.

Consequently, a rectangular wave guide is preferred whenever there is a chance that the polarization may be rotated in an uncontrolled fashion. Experience has shown that unavoidable irregularities, or deliberate distortions, such as bends, do tend to rotate the plane of polarization in circular wave guides to such an extent as to interfere seriously with the normal operation of the system. In general, therefore, wave guide components have been developed with rectangular guides. Best results are generally obtained when the proportions of the cross section lie in the vicinity of 1 by 2, with the larger inside dimension equal to about three-quarters of a wavelength. The microwave bridges discussed here are constructed with rectangular wave guides.




ELECTRIC FIELD CONFIGURATION IN CIRCULAR, SQUARE, AND OBLONG GUIDES

FIG. 1

## HYBRID DEVICES

Several hybrid devices have been developed which perform many important functions at microwave frequencies including uncoupling the receiver from the transmitter, isolating the local beating oscillator from the antenna, and that of a bridge circuit.

The performance of these devices is very similar to that of the hybrid coil, long familiar in telephone practice as a means for securing preferential isolation of circuits.

A hybrid device is shown in Figure 2 schematically as an eight terminal network. When a source of alternating electromotive force is connected across the terminals A, no signal appears at the terminals C, but transmission takes place freely between A and B and between A and D, with the input thus divided between equal loads placed at B and D. If the generator is transferred from A to C, no voltage can appear at A, from reciprocity considerations and again the input power is divided in some fashion between loads at B and D. It is generally desirable, moreover, that the terminals B and D be balanced with respect to each other, so that a generator applied at either of these points delivers power only to loads at A and C. In most cases the circuit is so proportioned as to bring about equal power division between the driven loads.

In constructions suitable for use at microwave frequencies, the terminals correspond to four appropriate transmission lines, not necessarily alike, emerging from the network. The loads

or generators attached to these lines are assumed to terminate each arm in its characteristic impedance. If this is not done, reflections from the loads will tend to upset the balance.

The elements comprising the hybrid network are presumed to be primarily reactive, so that internal dissipation is reduced to a minimum. The resistive component of the impedance viewed at any one of the emerging lines corresponds, therefore, almost solely to the loads in the two adjacent lines, transformed by the network. A properly compensated circuit is one which combines these loads so as to present at any arm an impedance equal as nearly as possible to the characteristic impedance of that arm.

Within the hybrid network the four emerging lines must be joined in some manner. Two distinct possibilities can be visualized. The terminal lines may all be connected to another transmission line or they may be joined directly without any intervening medium. Correspondingly, two classes of circuits possessing hybrid characteristics will be described: hybrid junctions or "magic tees" as shown in Figure 3 and hybrid rings or "rat-races" involving a ring or loop of line to which connections are appropriately made.

MAGIC TEE

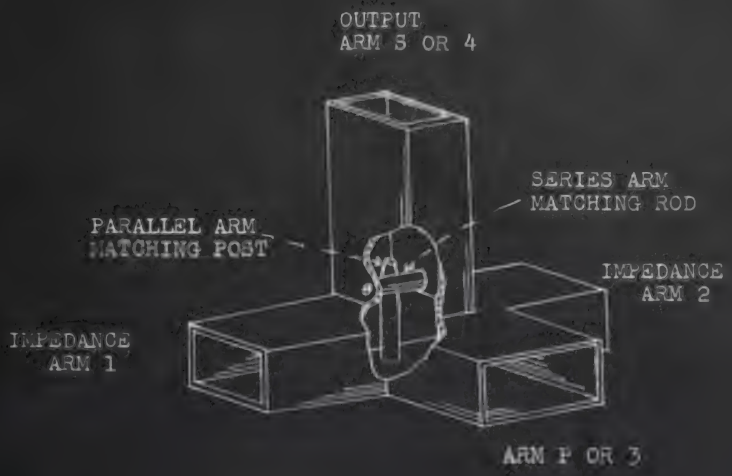
A hybrid network for microwaves can be secured in the form of a compound junction comprising a series connection and a parallel connection made to a guide at the same point on its longitudinal axis. Figure 3 shows a form of such a junction called the magic tee.

The fundamental property of the tee is that, if identical impedances are placed on the arms which are labelled impedance arms, then no power will be transmitted from the input to the output arm. This can be demonstrated by the use of symmetry arguments alone for the geometrical configuration shown in Figure 3 or by the analogy with optics. For a mathematical analysis see Appendix A.

This property suggests a method for comparing two impedances, since if the impedances are placed on the impedance arms and if there is no transmission through the tee, then the impedances are identical. When the impedances are not identical, the transmission through the tee gives a measure of the degree of difference of the impedances.

Basic Theory: The magic tee shown in Figure 3 is an example of the eight terminal network in Figure 2 and possesses the symmetry properties of that network.

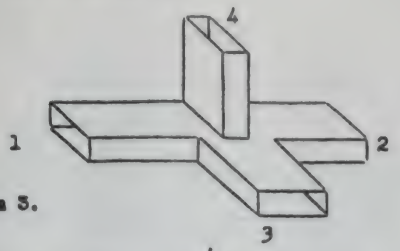
Some approximate field configurations are illustrated in Figure 4, which suggests how the geometric symmetry of the bridges produces the necessary electrical symmetries.




A PERSPECTIVE CUT-AWAY VIEW OF A MAGIC TEE USING WAVEGUIDE ARMS SHOWING "ROD" AND "POST" USED IN IMPEDANCE MATCHING.

FIG. 3

A bridge using waveguide arms




fed from Arm 3.

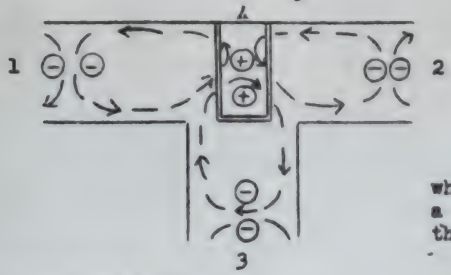
E ——→

H - - - →

⊖ into paper

⊕ out of paper

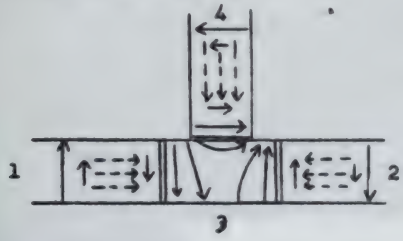
(a)




which cannot excite a wave in Arm 4 of the form:

and conversely, fed from Arm 4:

(b)




which cannot excite a wave in Arm 3 of the form:

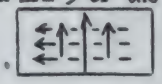


FIG. 4

Consider Figure 5 which shows the magic tee being used as a bridge to measure voltage standing wave ratio.

In general there will be waves traveling in both directions in all four arms of the tee. These waves will be described by their complex voltage amplitudes. Assume that the generator voltage is 1 + j0.

A double subscript will be used to designate voltage amplitudes. Thus $E_{3I}$ is the voltage amplitude of the wave traveling from 3 into the tee. $E_{I4}$ is the voltage amplitude of the wave traveling from the tee to 4. Since arm 2 is terminated with a matched load $E_{2I} = 0$.

Due to the symmetrical properties of the tee, if arm 1 is matched, there will be no power reaching 4 (the detector). However, if this arm is not matched, the reflected wave from 1 will cause power to be transmitted to the detector.

$E_4 = kE_{1I}$ where k is a function of frequency, and of the generator bridge, and detector impedance, but not of $Z_1$ (unknown impedance)

Let $Z_d$ = impedance of the detector

$R_d$ = real part of the $Z_d$

$P_d$ = power to the detector

then:

$$P_d = \left|\frac{E_4}{Z_d}\right|^2 R_d = \left|\frac{kE_{1I}}{Z_d}\right|^2 R_d$$

In arm 1:

$E_{1I} = A_1 E_{I1}$ where $A_1$ is the voltage reflection coefficient

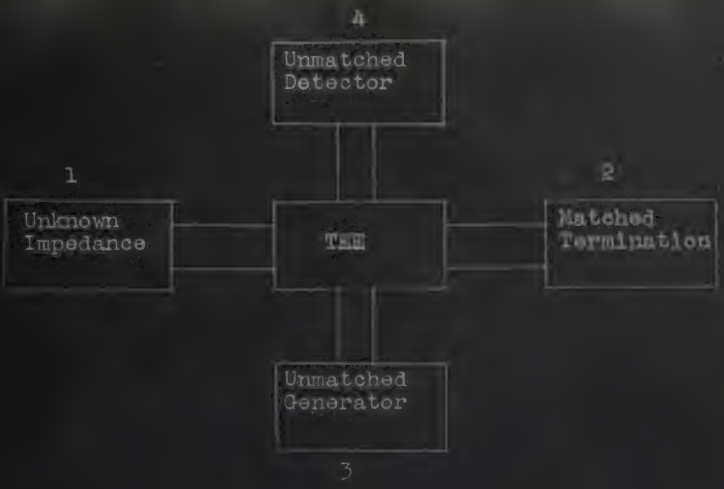



FIG. 5

Lines of electric intensity with a metal post

Circuit equivalent

Current flow with metal rod

Circuit equivalent

A "post" at upper left is equivalent to a shunt capacitor, while a "rod", lower left is equivalent to a shunt coil

FIG. 8

of the unknown impedance. Also, $E_{11}$ is the sum of contributions from two sources, the original wave in arm 1 from the generator ($E_o$) and a contribution ($cE_{1T}$) from the wave $E_{1T}$, part of which is reflected from the bridge.

i.e. $E_{11} = E_o + cE_{1T}$ , where c is the voltage reflection coefficient of the bridge looking in at arm 1 with all other terminals of the bridge loaded as shown in Figure 5.

Solving: $\frac{E_{1T}}{A_1} = E_o + cE_{1T}$

$$E_{1T} = \frac{A_1 E_o}{(1-A_1 c)} \quad \text{and}$$

$$(1) \quad P_d = c_2 |a_1|^2 \frac{A_1}{(1-A_1 c)^2}$$

The proportionality factor is usually determined experimentally in practice from (1)

$$A_1 = k\sqrt{P_d}\ |1 - A_1 c|$$

Now if $c = 0$ the power out is a true measure of $A_1$ or of the VSWR of the unknown impedance. Even if $c \neq 0$, the values of c and $A_1$ usually encountered do not give large deviations from the simple expressions. Figures 6 and 7 show the maximum error possible by neglecting the $|1 - A_1 c|$ term. This error is in the form of a phase error. At a given $\lambda$ and bridge adjustment, c will be constant. If $a_1$ is constant in magnitude and varies in phase, the term $|1 - A_1 c|$ will vary about the value 1. There will be two phases of the unknown for which there will be no error.

In addition to this phase error, there is usually an

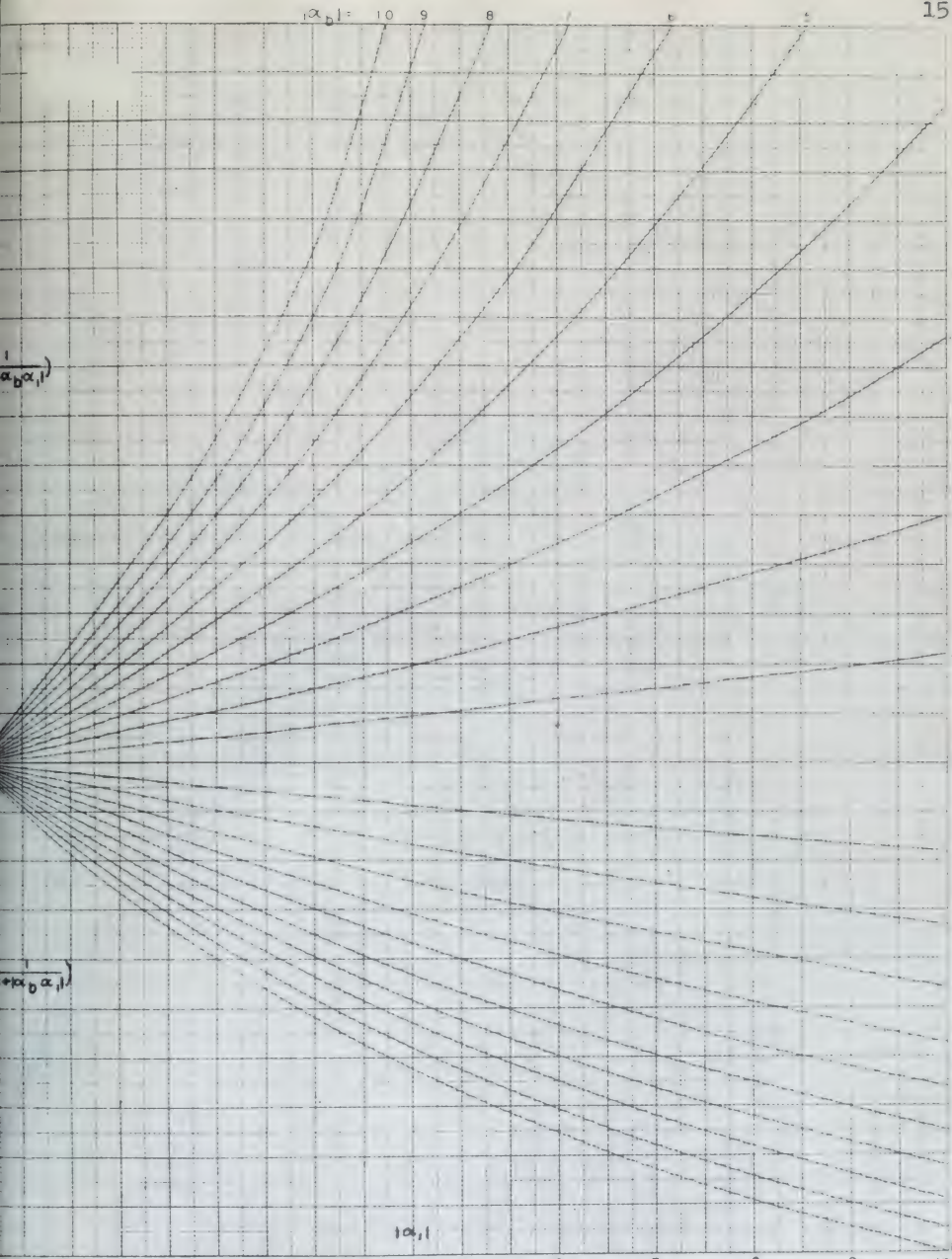

$|\alpha_0|$ = 10 9 8
$\frac{1}{\alpha_0\alpha_1}$
$+\alpha_0\alpha_1$
$|\alpha_1|$
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
4 3 2 1 0 1 2 3 4 5 6 7 8 9 10

FIG. 6

13248

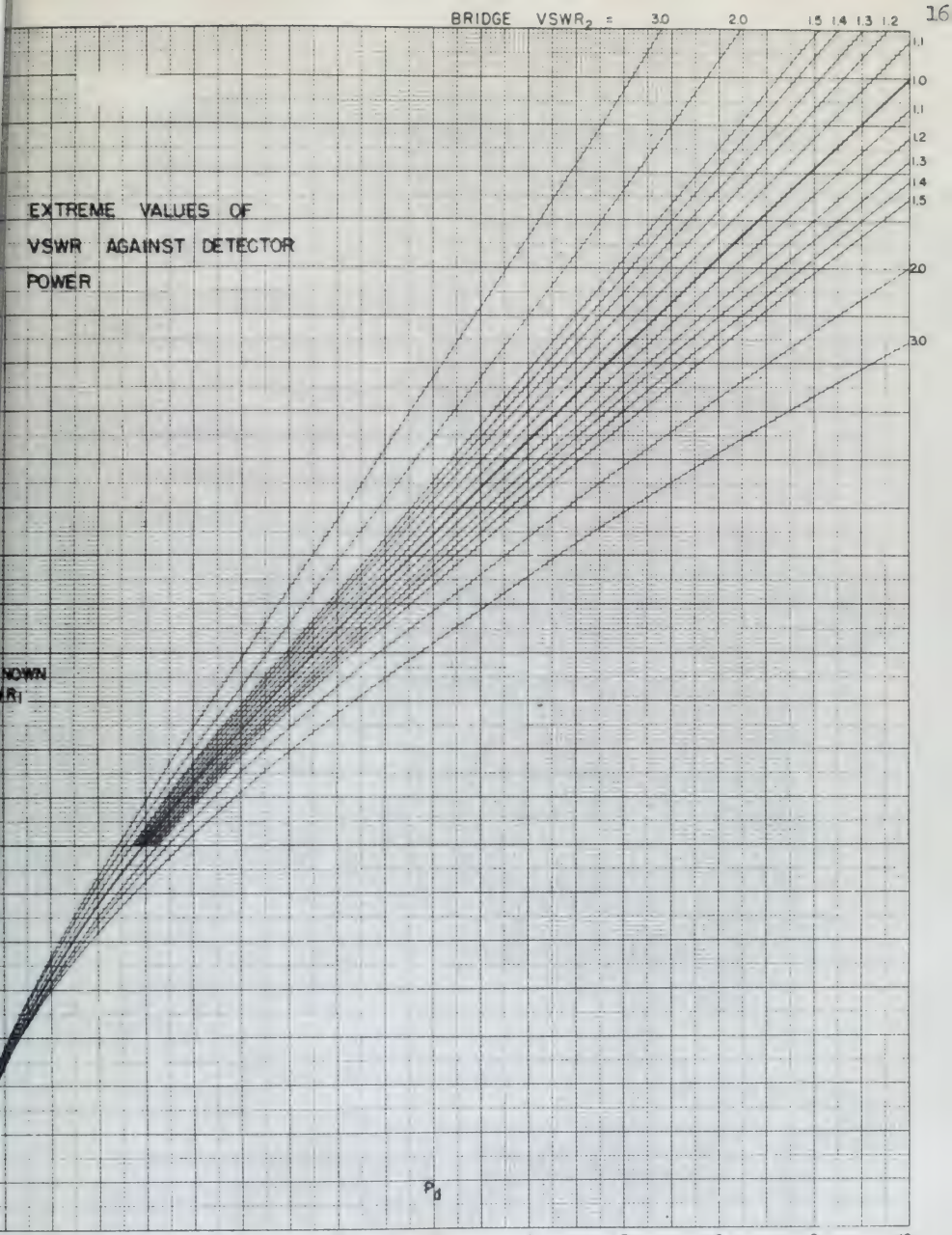

BRIDGE VSWR$_2$ = 3.0 2.0 1.5 1.4 1.3 1.2
1.1
1.0
1.1
1.2
1.3
1.4
1.5
2.0
3.0
EXTREME VALUES OF
VSWR AGAINST DETECTOR
POWER
NOWN
R$_1$
$P_d$
1 2 3 4 5 6 7 8 9 10

FIG. 7

error due to inherent unbalance of the tee, or, what amounts to the same thing, failure to use a perfectly matched termination on arm 2. This unbalance can be specified by giving the VSWR of the load on arm 1 which is necessary for balance. The error in measurement thus introduced behaves in the same way as a reflection in the line (of the same VSWR) in slotted section measurements.

For example, if a load of VSWR = 1.02 is required to balance the bridge, an unknown which apparently has a VSWR = $r$ as measured by the bridge, can actually have a VSWR lying between $r \times 1.02$ and $\frac{r}{1.02}$. This is in addition to any phase error.

Operating Procedure for Measuring VSWR: Usually the device to be tested must have a VSWR < N. The test equipment consists of the bridge element, a detector, a modulated CW oscillator, and an amplifier driving a meter. The test sample is placed on one impedance arm and a matched load on the other. To adjust the power level, a termination having a known VSWR = N is furnished. With this standard on the test arm, the level of the bridge is adjusted to give reference deflection on the meter. The test items can then be placed on the test arm in rapid succession and accepted or rejected. The setting of the bridge can readily be checked at any time against the standard mismatch.

The meter scale can be graduated to read directly in VSWR.

This procedure is of particular importance if it is necessary to tune the device tested to a match, as in the case of thermistor mounts. The device is adjusted while it is on the bridge,

and it is necessary only to obtain a minimum meter deflection.

The primary failing of this type of bridge is that it fails to give phase data. For many types of operations, however, this is no handicap.

Design Considerations: The features just described, depending upon the combination of geometrical symmetry with the polarization properties of rectangular wave guide, are inherent in the junction by itself. An important refinement is realized, however, when reactive tuning, as exemplified by the metal post and rod in Figure 3 is associated with the junction. These added elements improve the impedance match of the junction to each of the four arms, and thereby automatically bring about a high degree of balance between arms 1 and 2. Figure 8 shows the low frequency equivalents of the post and rod. The metal post concentrates the electric field between itself and the nearby wall of the guide, and is equivalent to a capacitance placed in shunt across a transmission line. The principal effect of the metal rod is to provide paths for conductive current flow between the connected walls of the guide, with a resulting increase in magnetic flux and flux linkages, and thus it is like an inductance in shunt across a line. Provided that reasonably good conductors have been selected for the post and rod, their resistance components can be neglected except in extreme cases.

Other tuning means, such as metal plates, can be used instead of the metal rod and post, but the latter is being widely used.

By a proper choice of tuning elements, effective neutralization of reflections can be obtained over a remarkably broad band of frequencies.

In practice magic tees at X-band, can be built without tuning devices to be balanced at a VSWR of 1.01. Such tees should be constructed of special accurate wave guide, with equal impedance arms and identical line fittings. An estimated accuracy of ± .001" in the alignment of arms 3 and 4 is required.

Another important characteristic of a magic tee is the impedance looking into the test arm (reflection coefficient = c). Since c is a function of the generator and detector impedances, it can be readily adjusted by tuners in either of these arms. For a broad band solution, however, it is desirable to use a matched generator and detector, and tune c by means of tuners placed close to the center of the tee junction. This problem is being worked on but a completely satisfactory solution is not available. The c can be reduced by inserting a small well-matched pad in the test arm without invalidating the theory, but at a cost of a loss in sensitivity of twice the attenuation of the inserted pad. This trick may be useful in measuring large VSWR's.

Relative Merits of the Tee Bridge and the Directional Coupler: Directional couplers can satisfy the requirements of a bridge element and are being used in this manner at present. See Appendix C for the mathematical analysis. Therefore, the relative merits of the tee as a bridge and the directional coupler should

be discussed.

The magic tee has the advantage on balance (or what is called directivity in a directional coupler) since it depends on geometrical symmetry alone. Careful design is necessary to produce a coupler with high enough directivity (about 30 db) to be useful for low VSWR's. The coupler can be trimmed with tuning screws to be accurately balanced but the frequency sensitivity will not be good. The ρ (reflection coefficient) is much better in the case of the directional coupler, since if a padded generator is used, the VSWR looking back from the test impedance can be very good indeed. This becomes more important in measuring large VSWR's. The directional coupler subtracts only a small fraction of the power in the line, which may be either an advantage or a disadvantage. This results in a loss in sensitivity, but it permits measurements to be made on a unit while in service.

Use of the Tee Bridge About A Point Other Than Match: Sometimes it is desired that a device have a definite impedance other than the line impedance. For this case, instead of placing a match on one of the impedance arms, one uses as a standard a unit having the desired impedance. Then, when the test item has the same impedance, the bridge is balanced. Instead of measuring the VSWR of the unknown, the VSWR meter now indicates the degree of difference between the unknown and the standard.

Measurement of Phase: The tee bridge does not measure phase in the rapid automatic way that it measures VSWR, but there are ways of using the bridge to get phaseswhich can compete with

slotted section methods. The tee bridge can be used with a calibrated variable impedance on one arm to make measurements, but the calibration and use of the variable impedance is laborious. This method is no more accurate than the method used to calibrate the variable impedance.

A subsidiary phase measuring device can be added to the tee bridge which consists of a sliding screw in the test arm. It is inserted to a fixed depth, and its position along the guide varied maxima and minima will be observed, on the indicating meter. These will be more pronounced than with a slotted section. A phase calibration can be made by placing an impedance of known phase on the test arm and observing the phase of the sliding screw. Unfortunately, if $c \neq 0$, there is a phase error, and the maxima and minima may not be evenly spaced. This is more pronounced at high VSWR. For very low VSWR this method may be preferable to the slotted section since the ratio of maximum to minimum will be large, especially if the sliding screw reflects about as much as the unknown. The sliding screw should be carefully built in a narrow slot. This screw is withdrawn completely while measuring VSWR.

## HYBRID RINGS OR "RAT RACES"

An examination of Figure 9 will show the principles involved in the construction of a hybrid network from a ring or loop of transmission lines. The cross section shown is in the electric plane of a wave guide ring with a straight guide connected to it symmetrically from the exterior. The electrical length around the ring is one and one-half wave lengths. Here, wavelength corresponds to the increased phase-velocity characteristic of a hollow pipe. It is assumed, and has been verified experimentally, that the electrical length around the ring is equal to the mean circumference.

When dominant waves are sent into the ring from the side arm, they will be split at the junction into two sets of waves of equal amplitude traveling around the ring in opposite directions. A pure standing wave will therefore be set up within the ring. This must be the case for no mechanism has been provided for dissipation within the ring other than the minor ohmic losses associated with wave transmission. The two sets of waves spreading out from the junction are 180 degrees out of phase, due to the nature of an electric-plane tee junction. When they reach the point diametrically opposite the junction, they have traversed paths of equal length and are still 180 degrees out of phase. This point is therefore a voltage minimum (zero). With this as a starting point, the standing wave within the ring can be plotted by marking off alternate voltage maxima and minima at 1/4 wave

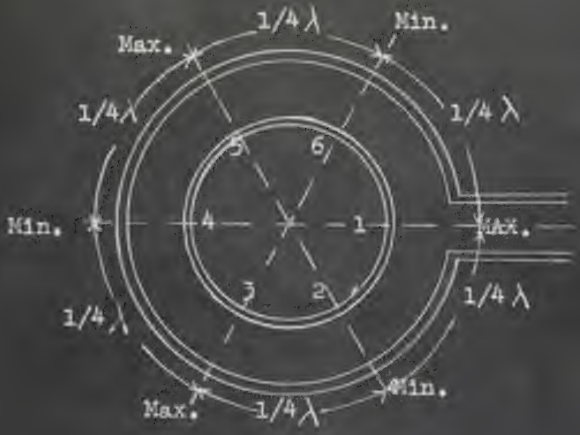

1/4 λ
Max.
Min.
1/4 λ
1/4 λ
5
6
Min.
4
1
Max.
3
2
1/4 λ
1/4 λ
Max.
1/4 λ
Min.

FIG. 9

3/4 λ
$K_o$
5
6
$K_o$
4
1
$K_o$
3
2
1/4 λ
1/4 λ
$K_o$
$K_o$
1/4 λ
$K_o$

FIG. 10

intervals.

If identical wave guides are connected symmetrically to the ring at points 2 and 4 so as to form additional electric-plane tee junctions, and if these side arms are terminated in their characteristic impedance , they will receive equal amounts of power which are large, since they are series connections at voltage minima. The amplitude of waves proceeding past 2 toward 3 will however still be equal to the amplitude of waves proceeding past 4 toward 3, since equal amounts of power are being extracted from two equal sets of waves. The arc 2 3 4 still contains, therefore, a pure standing wave. Since identical phase shifts have been introduced at 2 and 4 by the identical tee junctions, the position of this standing wave is unaltered, and there is still a voltage maximum at 3. A series connection to the ring at 3 will thus receive no power. Such a connection may be made by introducing a fourth electric plane junction at 3.

Figure 10 is a cross sectional view with series connections to the ring at 1, 2, 3, and 4. This represents a hybrid circuit with bidirectional characteristics. This may be shown by examining what happens when a generator is placed on arm 2 and appropriate loads in arms 1 and 3. At 5, the point diametrically opposite the driving point, the fields will interfere to produce a voltage minimum, and in the absence of arms 1, 3, and 4, there will be voltage maxima at 2, 4, and 6, and additional minima at 1 and 3. Arms 1 and 3 will thus receive equal amounts of power

without disturbing the position of the standing wave pattern in the arc 153 and the arm at 4 receives no power.

The ring described thus far is not the only structure of this type which functions as a hybrid circuit. If a series element of impedance Z on a transmission line of characteristic impedance $K_0$ is replaced by a shunt element of impedance $\frac{K_0^2}{Z}$ on one side of which is added a quarter wavelength of line and on the other side, a three-quarter wavelength of line, the entire performance of the circuit is unaltered. In this way the four series connections to the ring can be replaced, one by one, by shunt connections. This gives six distinct hybrid rings which are shown in Figure 11. These circuits apply equally well for low and high frequencies.

For microwave frequencies the practical realization of the circuits shown in Figure 11 calls for electric plane wave guide tee junctions as series connections and magnetic plane wave guide tee junctions or transformations to coaxial as parallel connections. In certain combinations there may be structural advantages in making the plane of the ring the magnetic plane. An interesting situation arises when square or circular wave guide is used, for here the circuits of Figure 11a and 11f can be included in the same structure. For one orientation of the polarization the ring lies in the electric plane; for another, the magnetic plane; and for any orientation in general, the input waves may be resolved into components parallel with and perpen-

(a)

(d)

(b)

(e)

(c)

(f)

FIG. 11

dicular to the plane of the ring for each of which a hybrid circuit is provided. In this case the matching of resistive impedances for both components becomes somewhat more involved.

The circumferential dimensions of hybrid rings can be changed in discrete steps without altering any of the circuit characteristics, by the application of either or both of the following rules.

(a) An integral number of wavelengths may be added to or subtracted from any arc (i.e. portion of ring between centers of adjacent connections).

(b) A pair of half wavelengths may be added to or subtracted from any two arcs.

These rules can be applied when it is desired to simplify the construction or to alter the structure to conform more readily to available space. Some of the dimensions in Figure 11 were arrived at using these rules. The proportions shown in Figure 10 conform in each case to the smallest ring in which all connections are separated by finite arcs. An additional pair of half wavelengths may be removed from each of the circuits of Figure 11 b and 11 c. Care must be taken, however, to preserve connections which are truly adjacent and which are not superposed symmetrically to the same point of the ring, if the properties of the original hybrid ring are to be preserved.

In general wave guide constructions are likely to demand an expansion from the dimensions of Figure 11. Rectangular wave

guides have become somewhat standardized upon an electric plane dimension only a little less than a half (free space) wavelength. Since the wavelength in the guide is seldom as great as two times the free space wavelength, the quarter (guide) wavelength spacings stipulated in Figure 11 correspond to extremely close spacings or sometimes even to overlapping of adjacent series connections. Greater difficulties will be experienced with adjacent connections in the magnetic plane. These troubles, when they arise, can be obviated by increasing the size of the ring according to the rules given. For example, the arcs in Figure 11 a may be increased to 3/4$\lambda$, 3/4$\lambda$, 3/4$\lambda$, 1 1/4$\lambda$.

It has been pointed out that for the wave guide form of the circuit in Figure 11a, the reactances associated with the junctions do not upset the performance because equal phase shifts are introduced as the waves travel past the two driven arms. The same arguments can be applied to the highly symmetrical circuits of Figure 11 d and 11 f. However, the situation is not so obvious with regard to the rings involving assymetrically disposed mixed connections, since different phase shifts may be expected at series and shunt branches. Consider the circuit of Figure 11 b. When this is driven from the shunt arm or from the opposite series arm, equal phase shifts take place at the two identical, adjacent series arms and balance is secured, with the dimensions as given. If the junction reactances are reduced or eliminated by symmetrical tuning, balance and equal power division will still be retained.

General considerations based on reciprocity may now be cited to show that this same tuning automatically brings about balance, equal division of power, and impedance match when the circuit is driven from either of the opposed series arms, even though the connections adjacent to these arms have such a totally different character. The same arguments can be applied to Figure 11 e. The completely asymmetrical circuit of Figure 11 c remains, then as the only one whose dimensions may perhaps require substantial alteration when appreciable phase shifts are involved.

For a mathematical analysis see Appendix B.

Design Considerations: There are undesired reflections within the ring which arise in two ways:

a. Those at the tee junctions to the ring caused by the abrupt geometry which characterizes these junctions. These may be reduced as in the magic tee by introducing in the vicinity of the junction appropriate reactive elements.

b. Those whose source may be traced to the inherent resistive mismatch which results from using wave guides of approximately the same size for the ring and all the arms. In this case a generator of impedance $K_0$ is connected by two loads essentially in series, each of which has an impedance approximately equal to $K_0$. This mismatch can be readily eliminated by altering the characteristic impedance of one or more of the wave guides involved.

Frequency Characteristics: Of the various electrical properties the balance between opposite arms is the most critical one. Consider what happens when in Figure 9 there are sent into the ring waves of a frequency different from that originally considered. There will still be cancellation of fields at point 4, but now the spacing in the standing wave pattern is changed and the voltage maximum is displaced from point 3. An arm attached at 3 will thus not be completely uncoupled from the arm at 1. This reasoning can be applied to the circuits shown in Figures 11a, 11d, and 11f, and also to those pairs of opposite connections in Figures 11b and 11e, which are balanced by virtue of two paths

around the ring which differ by a half wave length; that is, to all cases in which the opposite connections are both parallel or both series. A different situation exists when balance is obtained between a series connection and a shunt connection located at geometrically opposite points across the rings, since the standing wave pattern at the point diametrically opposite the feed point does not shift with frequency. This argument applies to Figure 11c and one pair of opposed arms in Figures 11b and 11e. When a high degree of balance is desired between one particular pair of arms over as broad a band as possible, the circuits of Figures 11b and 11e will be preferred, for they offer driven arms which are symmetrically connected.

If the loads attached to the arms of a hybrid ring are maintained nearly reflectionless over a band of frequencies, and if the junction reactances in the ring are effectively cancelled throughout this band, important variations in the power division or in the resistive matching ability of the network will not be expected. In general, limits to the usable band width will be set by deviations from balance or by inability to tune out satisfactorily the internal reflections associated with the junctions.

## APPLICATIONS

The very nature of a hybrid circuit is such as to suggest immediately numerous applications in duplexing. In Figure 2, for example, the following connections may be made: at A, a transmitter; at B, a transmission line or antenna; at C, a receiver; at D, a dummy load. Of the power sent into the network from the transmitter, half will be sent out the line or antenna, half will be dissipated in the dummy load, and none will appear in the receiver. Signals entering the network from the line or antenna will be divided equally between the receiver and transmitter. There will therefore be a 3-decibel loss in reception and in transmission, or the total round trip loss of 6 decibels often associated with hybrid devices. This duplex system still compares favorably with the two simplex systems necessary to accomplish the same result, for the total area of two similar antennas associated with the simplex systems may be devoted to a single antenna with a 3 decibel increase in gain, or in the case of transmission lines, the two lines needed in simplex operations may be replaced by a single improved line, so that again the 6 decibel round-trip loss is recovered. This loss can also be reduced materially by the appropriate employment of active circuit elements, such as gas-discharge switches.

Another important field of application occurs in connection with balanced mixers and converters. As an interesting preliminary to this subject, consider the following argument. In

Figure 2 let the terminals B and D be joined externally at a junction E. This junction may be so located that when power is introduced at A, the equal amounts of power delivered to B and D reach E in phase. All of the power from A will therefore be developed in the appropriate load at E. Since A and E are completely coupled, the terminals C, by reciprocity, have no connection with either A or E, and hence play no part in the circuit. Thus, it is not possible to combine the two radio-frequency voltages to drive a single load and to avoid the three decibel loss accompanying the use of a dummy load, without at the same time destroying the hybrid properties of the circuit.

It is possible however to avoid a loss, when detection is involved, by combining detected signals. It is this principle which is involved in the use of a hybrid network for a balanced converter in the manner represented schematically in Figure 12. The intermediate frequency voltages appearing at B' and D' must be combined with due regard for their relative phase. While it is important that there is no loss in signal power over that normally associated with detection, this in itself is scarcely a reason for preferring the balanced converter. Its advantages however are numerous including the isolation between the signal and and local beating oscillator and the elimination of noise contributions from the local oscillator. This has indeed been an application in which these hybrid circuits have been extensively used in microwave systems. For this purpose the hybrid junction or

in microwave systems. For this purpose the hybrid junction or

magic tee has been chosen, for reasons of compactness, convenient geometry, and broad frequency response.

One of the more important uses of these hybrid circuits in impedance bridges may be exemplified by the following method of measurement of the band pass characteristic of low-QTR tubes. Assume that a magic tee is used and arrange it as follows: Test piece or unknown impedance $Z_x$ on arm 1, generator on arm 3, detector on arm 4, and a characteristic impedance $Z_0$ on arm 2. The power input arm 4 is a measure of the magnitude of the voltage reflection coefficient of the unknown impedance $Z_x$. The error encountered depends on the match of the detector and generator and the mechanical asymmetry in the magic tee. If this tee is used with a modulated signal source and a crystal detector, the tuning of a device which must be matched at one frequency or which must have a reflection coefficient less than a certain value at the one frequency is simple. The tuning of the device is adjusted for minimum power in the output arm. With a perfect magic tee this will be zero for match; otherwise, it will depend on the desired reflection coefficients.

With appropriate precautions it is possible to use a single magic tee with several input frequencies. By the use of a corresponding number of local oscillators in a circuit similar to a spectrum analyzer, the power from the output arm may be displayed on an oscilloscope in the form of pips, one pip for each frequency. If three frequencies are used, the behavior of the reflecting element

at the midband and band-edge frequencies is easily determined, and may be observed at a glance. If the device is calibrated with a reflection of known magnitude, quantitative data may be obtained. For example, if it is desired to check a resonant window or a single resonant element, such as is used in a band pass TR tube, the three frequencies corresponding to midband and the two bandedge frequencies are set on the bridge. The element under test, backed by a matched load, is put on one arm of the magic tee. The heights of the pips then give the desired information. The tuning of an element can be accomplished by adjusting for a symmetrical pattern. In addition to a quick examination of the characteristics of single elements, it is also possible to tune two-element bandpass TR tubes and to check the overall response curve, that is, the standing wave ratio looking through the tube into a matched load at three different points in the passband.

This type of bridge is excellent for production checking of components, such as the windows of the bandpass TR tubes, especially for tuning, since the necessary symmetry of the Q-curve for the proper frequencies can be noted at a glance. Any necessary changes on transmission-resonant elements, such as grinding the glass in the windows or filing metal in other types of elements, may be quickly checked between steps by noting the change in the pips.

Beyond these and other applications for hybrid circuits in microwave systems, there are numerous uses in laboratory measure-

ments. These will arise whenever it is necessary to establish directional balance or to make special use of the phase properties which characterize these circuits.

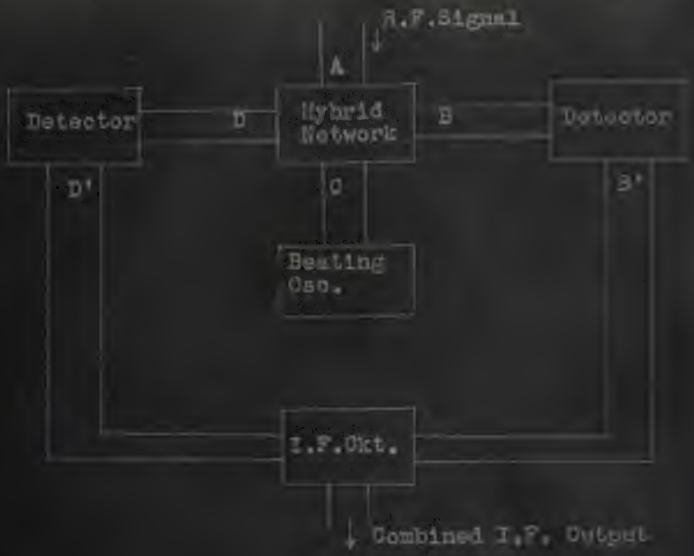

R.F.Signal
A
Detector
D
Hybrid Network
B
Detector
D'
C
B'
Beating Osc.
I.F.Ckt.
Combined I.F. Output

FIG.12

FIG. 13

## SIX-ARM MICROWAVE BRIDGE

A six-arm wave guide structure shown in Figure 13 has been developed which is a true microwave equivalent of a wheatstone bridge. The other microwave devices which have been discussed and which measure impedances by comparison are not actually bridges in the sense that one uses the term at low frequencies. To be a true bridge of the wheatstone type, they must obey the equation

$$AB = CD$$

$$(1) \quad \text{or} \quad B = \frac{C}{A} D$$

Where A, B, C, and D are impedances, three of which are known and the other unknown. The magic tee and retraces are not true bridges, for they obey the equation

$$(2) \quad A = B$$

It is necessary in (2) to provide an adjustable impedance equal exactly to the unknown. Thus, the standard impedance will require both a variable resistance and a variable reactance. At microwave frequencies a variable resistance is difficult to provide. This is a disadvantage of using a hybrid ring or junction. With the true wheatstone bridge, however, any impedance may be matched by using a fixed resistance and variable reactances. Both of these are simple to make at microwave frequencies. As an example, let B be the unknown. Then if A and C are variable reactances consisting of shorted sections of transmission lines of variable length,

$$\frac{dy}{dx} = \frac{y}{x}$$

the ratio C/A can be adjusted to have any real value desired. D consists of a perfectly terminated line in series with a variable reactance, which is another shorted-section line. By adjusting the latter, D can be made to have any arbitrary phase angle. Therefore, D can be adjusted to have the phase angle of the unknown and the ratio C/A can be adjusted to give the correct magnitude. The value of the unknown B, then, is determined from the physical lengths of stubs A, C, and D.

The bridge shown in Figure 13 satisfies 1. There are six terminal pairs; the four impedance arms, and an input and output arm. This bridge is made by joining six pieces of rectangular wave guides at a symmetrical junction. Opposite pairs of arms, such as A and C or B and D, lie on common axes, but are rotated 90° with respect to each other.

This bridge was first suggested by R. Dicke, formerly of the M. I. T. Radiation Laboratory. It appeared plausible that the device in Figure 13 was a true Wheatstone bridge from consideration of the geometry and the way in which the various guides are connected. The designer then proceeded to show analytically that this device is the equivalent circuit.

The procedure was to select sets of waves in the structure, such that the field distribution due to any arrangement of terminating admittances and sources can be represented by linear combination of these waves. Thus, upon consideration of these waves and the symmetry properties of the bridge, relations between

the input admittance at one terminal pair and all the other five terminating admittances can be found. The waves that are set up are for the lowest mode in the wave guide only; that is, it is assumed that one is measuring the field sufficiently far from the junction of the guides so that all the higher modes which are present in the fringing fields at the junction are attenuated. The effect of the junction appears in the equation for the bridge in the form of equivalent lumped susceptances at suitably chosen reference planes in the guides. The expression for the admittance relation among the six arms is:

$$\begin{aligned} 3 - & Y_x Y_{x'} Y_z + Y_x Y_{x'} Y_{z'} + Y_x Y_{x'} Y_y + Y_x Y_{x'} Y_{y'} + Y_y Y_{y'} Y_z + \\ & Y_y Y_{y'} Y_{z'} + Y_y Y_{y'} Y_x + Y_y Y_{y'} Y_{x'} + Y_z Y_{z'} Y_x + Y_z Y_{z'} Y_{x'} + \\ & Y_z Y_{z'} Y_y + Y_z Y_{z'} Y_{y'} + Y_x Y_y Y_z + Y_{x'} Y_y Y_z + Y_x Y_y Y_{z'} + \\ & Y_{x'} Y_y Y_{z'} = 0 \end{aligned}$$

where $Y_x = Y_x + jb$

$Y_{x'} = Y_{x'} + jb$ etc.

$Y_x$ is the terminating admittance on arm x etc. This is the equation of a Wheatstone bridge with fringing susceptance of magnitude jb across each terminal. For the derivation of 3 see Bibliography No. 5.

In terms of this general admittance relation, the condition for a balanced bridge is that the input admittance is independent of the detector admittance, and a relation of the form 1 will fulfill this requirement. This balance equation, as modified by the fringing susceptances and using a slightly different notation,

can be written

$$4 - (Y_a + jb)(Y_b + jb) = (Y_c + jb)(Y_d + jb)$$

The equivalent circuit is shown in Figure 15.

Design Considerations and Bridge Characteristics: The ends of the wave guides which come together at the junction have triangular tongues, as shown in Figure 14, cut in each side, and when the six guides are placed together, these tongues mate to give the correct orientation for all the guides.

As is always the case for such equivalent circuits to be applicable, the admittances or impedances must be referred to certain special planes in the various arms of the structure. In this case, the equivalent plane for any of the bridge arms is the position of a shorting plunger which will cause a short to appear in the corresponding branch of the equivalent circuit. It is clear that, if the admittances $Y_a$, etc., which are measured at such equivalent planes satisfy an equation of the form 4, they will no longer satisfy it after they have been transformed an arbitrary distance down the line from the place at which they were originally defined. These equivalent planes can be located by moving shorts in two adjacent arms such as A and C, until the standing-wave ratio in the input circuit is infinite. In this case, the positions of the shorts determine the equivalent planes in A and C. The position of the node in the standing wave in the input arm also defines the equivalent plane in that arm.

There is a more practical method for locating the equivalent plane which depends on the fact that when shorts are located at the equivalent planes of A and C, corresponding to infinite admittances in the branch A-C, then the output of the detector

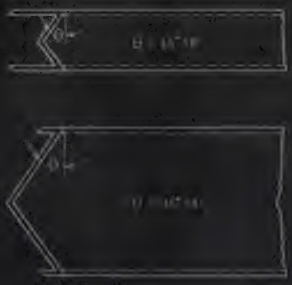


FIG. 14

FIG. 15

in turn, independent of variations in $Y_b$ and $Y_d$. The admittances $Y_b$ and $Y_d$ can be modulated by setting rotating electric fans near the open end of arms B and D, resulting in a modulated detector output. The short positions in A and C are then varied until the detector output is reduced to zero, which determines the equivalent planes for these arms, and also in the input arm. Planes in the other arms are located in the same way, by choosing different input arms.

In use as a bridge a calibrated movable short is placed on arm D in such a position that normalised admittance 1 + jY appears at the equivalent plane. Movable shorts are attached to arms A and C and the admittance to be measured is attached to arm B. Then

$$Y_b = \frac{Y_a}{Y_c} Y_d \text{ if fringing}$$

susceptance due to asymmetries in construction has been eliminated. This fringing susceptance is frequency-dependent but can be eliminated at any particular frequency by inserting a capacitive screw at the position of the equivalent plane or a quarter wavelength away, depending on the sign of jb.

Once properly adjusted, the bridge is simple to use, and measurements can be made rapidly. The unknown impedance is determined in terms of three physical lengths, the distances of the shorts from their correct reference planes, which is a very simple standard and is independent of detector calibration. The accuracy of the measurement is equivalent to that of a good standing-wave

detector. The use of the three physical lengths to specify the unknown impedance can be facilitated with the aid of simple formulas or charts. Although, in the current version of the bridge described here, the fringing susceptance is eliminated for only one frequency, it should be possible by conventional broad banding techniques to make the device quite broad band, so that the fringing susceptances are almost zero over a considerable range of frequency.

Four-Terminal Application: Since the six-arm bridge is the exact equivalent of a Wheatstone bridge, it can be used as a four terminal lattice section in microwave filter design or in similar applications. Most microwave filter structures used up to now have been the equivalents of ladder sections. A lattice section is the most general form of four-terminal network and offers much greater flexibility in design.

As an example, the transfer constant and image impedances can be separately adjusted. Also, any physical four-terminal network can be synthesized by means of lattice sections, while this is not always true for ladder sections.

## CONCLUSION

The magic tee, the directional coupler, and the hybrid ring may all be used in microwave bridges. However, the magic tee is chosen in most applications, principally, for reasons of compactness, convenient geometry, and broad frequency response.

The magic tee has the advantage of balance on the directional coupler since in the tee balance depends on geometrical symmetry alone. The directional coupler does, however, permit measurements to be made on a unit while in service.

The band width over which the hybrid ring operates differentially is inherently narrower than that of the magic tee. The rings do have considerably higher power capacity though.

The six-arm microwave bridge is an improvement for laboratory measurements. With this bridge it is possible to measure any impedance to about the same accuracy as with a standing-wave detector. A valuable feature of this instrument is that the standard impedances required are three variable reactances (movable shorting plungers) and a $Z_0$ termination. The data are obtained in the form of three lengths, the positions of the movable shorts.

The disadvantages of this Wheatstone equivalent bridge is narrow band width, and difficulties in construction, resulting in asymmetries and requiring tuning devices.

There are many applications for these bridges including production checking of bandpass characteristics, and in mixers, in converters, and in duplexers.

## APPENDIX A

### Analysis of Tee's

The following analysis uses the symmetry properties of solutions of Maxwell's equations in deriving results. These results are expressed in terms of a small number of parameters, which could be determined theoretically from an exact solution of Maxwell's equations, or which may be regarded as parameters in equivalent circuits for the various systems, which can be determined by appropriate experiments. The general method is to set up certain standing-wave solutions of Maxwell's equations, and to express the general solution as a linear combination of these independent standing waves.

If Maxwell's equations inside the cavity of the tee are solved, for a given frequency, an infinite variety of solutions are found, unless boundary conditions at the outputs are specified. If only those solutions in which there is real propagation in at least some output are considered, we are able to write all such solutions as linear combinations of a finite number of solutions. In the case of the tee, there are four such solutions.

Each of these solutions will have complicated behavior in the central part of the cavity, which we will consider only as to its symmetry. But in the wave guide, however, the field, which can be written as a superposition of all normal modes of the guide, becomes simplified by the attenuation of all those modes which are not propagated. We shall assume that the frequency is

such that only the dominant mode is propagated in the output guides; thus at some distance the field reduces to this mode, and can be characterized entirely by two constants, a phase and an amplitude, or a current and a voltage.

Now referring to Figure 3, let us choose symmetrically placed planes in arms 1 and 2 in an arbitrary manner. Four standing waves are then set up, two antisymmetric, two symmetric. One of each will have V = 0 at the planes of reference in 1 and 2, and one will have i = 0 at the planes. Now let us choose the plane of reference in 3 so that the antisymmetric function which has zero voltage across the reference planes in 1 and 2 likewise has zero voltage across the reference plane in 3. Then, let us choose the plane of reference in 4 so that the symmetric function which has zero voltage across the planes in 1 and 2 has zero voltage across the plane in 4. Thus, we have the following:

1. $V_{1a} = V_{2a} = V_{3a} = V_{4a} = 0$; $i_{1a} = -i_{2a}$; $i_{4a} = 0$

2. $V_{1b} = -V_{2b}$; $V_{4b} = 0$; $i_{1b} = i_{2b} = i_{4b} = 0$

3. $V_{1c} = V_{2c} = V_{3c} = V_{4c} = 0$; $i_{1c} = i_{2c}$; $i_{3c} = 0$

4. $V_{1d} = V_{2d}$; $V_{3d} = 0$; $i_{1d} = i_{2d} = i_{3d} = 0$.

The voltages and currents at the various reference planes are then given by:

5. $V_1 = C_b V_{1b} + C_d V_{1d}$; $V_2 = -C_b V_{1b} + C_d V_{1d}$

6. $V_3 = C_b V_{3b}$; $V_4 = C_d V_{4d}$

7. $i_1 = C_a\, i_{1a} + C_c i_{1c}$; $i_2 = -C_a i_{1a} + C_c i_{1c}$

8. $i_3 = C_a i_{3a} + C_b i_{3b}$; $i_4 = C_c i_{4c} + C_d i_{4d}$

Where $C_a$, $C_b$, $C_c$, and $C_d$ are appropriate complex coefficients used when building up general solutions by superposing solutions.

From the above, we can set up two equations.

9. $$\frac{i_3}{V_3} = \frac{i_{3b}}{V_{3b}} + \frac{C_a\, i_{3a}}{C_b\, V_{3b}} = \frac{i_{3b}}{V_{3b}} + \frac{i_{3a}\, V_{1b}}{V_{3b}\, i_{1a}}\left(\frac{i_1 - i_2}{V_1 - V_2}\right)$$

10. $$\frac{i_4}{V_4} = \frac{i_{4d}}{V_{4d}} + \frac{C_c\, i_{4c}}{C_d\, V_{4d}} = \frac{i_{4d}}{V_{4d}} + \frac{i_{4c}\, V_{1d}}{V_{4d}\, i_{1c}}\left(\frac{i_1 + i_2}{V_1 + V_2}\right)$$

Let

11. $Y_3 = Z_o\,(i_3/V_3)$

12. $Y_4 = Z_o\,(i_4/V_4)$

13. $Y_1 = Z_o\,(i_1/V_1)$

14. $Y_2 = Z_o\,(i_2/V_2)$

This leads to

15. $$Y_3 = -jb_3 - a_3\left(\frac{Y_1V_1 - Y_2V_2}{V_1 - V_2}\right)$$

16. $$Y_4 = -jb_4 - a_4\left(\frac{Y_1V_1 + Y_2V_2}{V_1 + V_2}\right)$$

Where

17. $$-jb_3 = \frac{i_{3b}}{V_{3b}}\, Z_o; \quad -jb_4 = \frac{i_{4d}}{V_{4d}}\, Z_o$$

18. $$a_3 = -\frac{i_{3a}}{V_{3b}}\frac{V_{1b}}{i_{1a}} \qquad a_4 = \frac{-i_{4c}}{V_{4d}}\frac{V_{1d}}{i_{1c}}$$

From 15 and 16

$$19.\quad Y_1Y_2 + \frac{(Y_3 + jb_3)(Y_4 + jb_4)}{a_3 a_4} + \frac{1}{2}(Y_1 + Y_2)\left[\frac{(Y_3 + jb_3)}{a_3} + \frac{(Y_4 + jb_4)}{a_4}\right] = 0$$

This is one form of the admittance equation for the Y.

In 19 if $Y_1 = Y_2$ we have

$$20.\quad \left(Y_1 + \frac{Y_3 + jb_3}{a_3}\right)\left(Y_1 + \frac{Y_4 + jb_4}{a_4}\right) = 0$$

One or the other factor must be zero. Thus, suppose the first factor is zero; we then have for the admittance seen looking into the output 3

$$21.\quad -Y_3 = jb_3 + a_3Y_1$$

Thus, the admittance is independent of $Y_4$. The reason is that with equal loads in 1 and 2, no power fed into output 3 proceeds to 4; it is all divided equally between 1 and 2. Hence, there is no effect of the admittance in 4 on the admittance as seen in 3. Similarly, if the power is fed into 4, none will proceed into 3, and we have

$$22.\quad -Y_4 = jb_4 + a_4Y_1$$

These results suggest an experimental method of determining the constants of the Y. Remembering that any symmetrically chosen pair of planes in one unit can simply be taken as planes of reference in linear outputs, choose them, and then place short-circuiting plungers on these planes, so that $Y_1 = Y_2 = \infty$. Then feed power into output 3. Then if we should have $-Y_3 = \infty$ or an infinite standing wave ratio, with the plane of reference in 3 at the standing wave minimum. Then remove the plungers and

match 1 and 2, so that $Y_1 = Y_2 = 1$. Now feed power into 3 and observe the admittance $-Y_3 = jb_3 = 2a_3$, from which $b_3$ and $a_3$ can be found by standing wave measurement. By similar methods in output 4, $b_4$ and $a_4$ are found.

Now to consider the question of power flow, let P be the power flow out an output and $b'$ be the real part of Y. Then the following is true

23. $P_1 : P_2 : P_3 : P_4 = b_1' |V_1|^2 : b_2' |V_2|^2 : b_3' |V_3|^2 : b_4' |V_4|^2$

From 6, 15, and 16

24. $V_3 = \frac{V_{3b}}{V_{1b}} \left( \frac{V_1 - V_2}{2} \right)$; $V_4 = \frac{V_{4d}}{V_{1d}} \left( \frac{V_1 + V_2}{2} \right)$

The only non-trivial relations of $\frac{V_{3b}}{V_{1b}}$ and $\frac{V_{4d}}{V_{1d}}$ are

25. $\frac{V_{3b}}{V_{1b}} = \sqrt{\frac{2}{a_3}}$ ; $\frac{V_{4d}}{V_{1d}} = \sqrt{\frac{2}{a_4}}$

From 23, 24, and 25

26. $P_1 : P_2 : P_3 : P_4 = b_1' \left| \frac{Y_3 + j b_3}{a_3} + Y_2 \right|^2 : b_2' \left| \frac{Y_3 + j b_3}{a_3} + Y_1 \right|^2 :$

$\frac{b_3'}{2a_3} |Y_2 - Y_1|^2 : \frac{b_4'}{2a_4} \left| 2 \left( \frac{Y_3 + j b_3}{a_3} \right) + Y_1 + Y_2 \right|^2$

27. $P_1 : P_2 : P_3 : P_4 = b_1' \left| \frac{Y_4 + j b_4}{a_4} + Y_2 \right|^2 : b_2' \left| \frac{Y_4 + j b_4}{a_4} + Y_1 \right|^2 :$

$\frac{b_3'}{2a_3} \left| 2 \left( \frac{Y_4 + j b_4}{a_4} \right) + Y_1 + Y_2 \right|^2 : \frac{b_4'}{2a_4} |Y_2 - Y_1|^2$

Using 19 two other equations of this form may be established, all equivalent to each other. We will use 27 for the case where $Y_1$

is nearly equal to $Y_2$ and power is being fed into the output 3. This is approximately the situation described in 21. For $Y_1 = Y_2$ equation 27 reduces to

28. $$P_1 : P_2 : P_3 : P_4 = b_1' \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : b_1' \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : \frac{2b_3'}{a_3} \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : 0$$

Taking the real part of 21, we have in this case $-b_3' = a_3 b_1'$. Thus in this case

29. $P_1 : P_2 : P_3 : P_4 = 1 : 1 : -2 : 0$ or the power flowing into output 3 is equally divided between 1 and 2, none flowing out 4, as we have mentioned earlier. If $Y_1$ is not exactly equal to $Y_2$, however, 27 serves to give the power flowing out of output 4, and to show that it is proportional to the square of the magnitude of $Y_2 - Y_1$.

Equation 26 is useful if $Y_1$ is nearly equal to $Y_2$, but power is fed into output 4.

In the equations derived above, there is not complete symmetry between the outputs 1 and 2 on the one hand, 3 and 4 on the other, the admittances $Y_1$ and $Y_2$ are balanced by the more complicated quantities $\frac{Y_3 + jb_3}{a_3}$ and $\frac{Y_4 + jb_4}{a_4}$. In order to secure complete symmetry, we must have $b_3 = b_4 = 0$, $a_3 = a_4 = 1$. This can be done by inserting irises in outputs 3 and 4. The balancing quantities above will then be replaced by $Y_3$ and $Y_4$ respectively in 19, 26, and 27, and there will be complete symmetry between

the two sets of outputs. Equation 21 will then become $-Y_3 = Y_1$, showing that if $Y_1 = Y_2 = 1$, so that the two terminals 1 and 2 are matched, we shall see a match looking into output 3. A Tee with these characteristics is generally called a Magic Tee.

## APPENDIX B

### Analysis of Hybrid Rings

This analysis is on hybrid rings of the series type. The ring is considered as a continuous line closed on itself. The sections between series taps are then treated as being made up of integral single or multiple quarter wave line sections. Lattice or equivalent T sections are derived for 1, 2, 3, and 4 quarter wavelength sections, ignoring line dissipation. Then, equivalent mesh networks are drawn from these equivalences. The mesh networks are then solved by determinants. To study some effects of frequency shift off the design center, where the mean periphery of the ring departs from an exact integral number of quarter wavelengths, the increments in the element values for a quarter-wave equivalent T section are calculated and utilized. The example studied is a ring of 1 1/2 $\lambda$ mean perimeter with three and four taps.

In this analysis $Z_0$ represents guide impedance and $Z_T$ represents annulus impedance. These rings may be considered on the basis that the annular slot is a transmission line, of characteristic impedance $Z_T$ and propagation constant $\Gamma$. This transmission line is closed upon itself. Series connections are made by the wave guide connections. The outlets of the wave guide are assumed to present wave guide characteristic impedance to the ring of $Z_0$. This may be shown as in Figure 16.

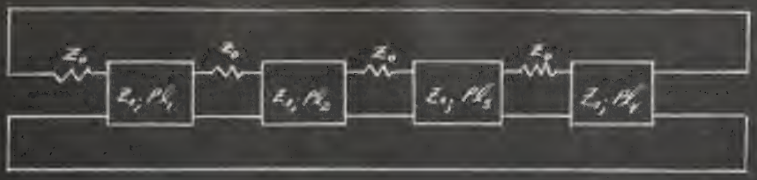



FIG 16

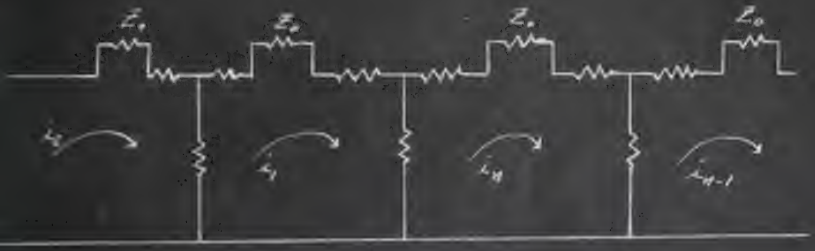



FIG 17

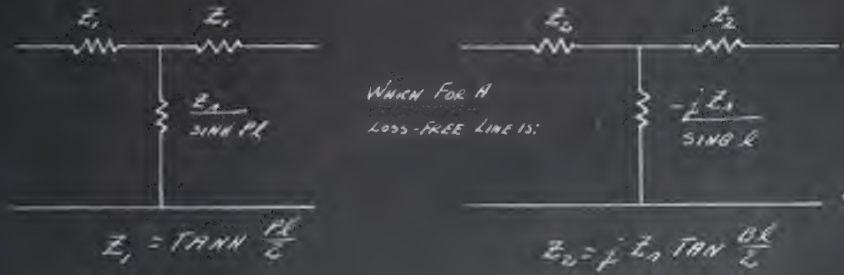



FIG 18

Now consider the line as made up of the appropriate number of quarter-wave sections, with series taps as shown in Figure 17.

The shunt impedances are identical; call each X. The series impedances are made identical by first assuming a tap at each quarter-wave junction. Let each series leg be S. Then, for a 2 1/2 wave length ring or N = 10, the (skew symmetrical) circuit determinant is:

(1) $D_{10} =$

$$\begin{vmatrix} S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X \\ -X & S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -X & S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -X & S+2x & -X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -X & s+2x & -X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X & S+2x & -X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -X & S+2x & -X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x & -X & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x & -X \\ -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x \end{vmatrix}$$

For the case of an exact number of quarter wave lengths around the ring, all $X_{1-n} = jZ_r$ and all $S_{1-n} = Z_o + 2jZ_r$.

Then, the determinant becomes

$$
(L)\quad D_n^{(2)} = \begin{vmatrix}
2\cos\theta & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
-1 & 2\cos\theta & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2\cos\theta & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2\cos\theta & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2\cos\theta & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2\cos\theta & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2\cos\theta & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2\cos\theta & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2\cos\theta & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2\cos\theta
\end{vmatrix}
$$

$$(2)\quad D'_{10} = \begin{vmatrix} Z_o & jZ_T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_T & a_1 \\ jZ_T & Z_o & jZ_T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 \\ 0 & jZ_T & Z_o & jZ_T & 0 & 0 & 0 & 0 & 0 & 0 & a_3 \\ 0 & 0 & jZ_T & Z_o & jZ_T & 0 & 0 & 0 & 0 & 0 & a_4 \\ 0 & 0 & 0 & jZ_T & Z_o & jZ_T & 0 & 0 & 0 & 0 & a_5 \\ 0 & 0 & 0 & 0 & jZ_T & Z_o & jZ_T & 0 & 0 & 0 & a_6 \\ 0 & 0 & 0 & 0 & 0 & jZ_T & Z_o & jZ_T & 0 & 0 & a_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & jZ_T & Z_o & jZ_T & 0 & a_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_T & Z_o & jZ_T & a_9 \\ jZ_T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_T & Z_o & a_{10} \end{vmatrix}$$

For a 1 1/2 $\lambda$ ring n = 6 and the system becomes

$$(3)\quad D_6 = \begin{vmatrix} Z_o & jZ_T & 0 & 0 & 0 & +jZ_T \\ +jZ_T & Z_o & +jZ_T & 0 & 0 & 0 \\ 0 & +jZ_T & Z_o & +jZ_T & 0 & 0 \\ 0 & 0 & +jZ_T & Z_o & +jZ_T & 0 \\ 0 & 0 & 0 & +jZ_T & Z_o & +jZ_T \\ +jZ_T & 0 & 0 & 0 & +jZ_T & Z_o \end{vmatrix}$$

For the study of the effects occurring if we move off the design center, the individual T's may be changed by a length $l = \frac{\lambda}{4} \pm \frac{\lambda}{N}$. Assuming where is no line dissipation and N large, so $\frac{\lambda}{N} << \lambda$, then each series arm is:

$$(4)\quad j Z_1 \tan\left[\frac{2\pi}{\lambda}\left(\frac{\ell}{2}\right)\right] = j Z_1 \tan\left[\frac{2\pi}{2\lambda}\left(\frac{\lambda}{4} \pm \frac{\lambda}{N}\right)\right] =$$

$$j Z_1 \tan\left[\frac{\pi}{4} \pm \frac{\pi}{N}\right] = j Z_1 \frac{1 \pm \tan\frac{\pi}{N}}{1 \mp \tan\frac{\pi}{N}} \doteq$$

$$j Z_1 \left(1 \pm \frac{2\pi}{N}\right) = j Z_1 \left(1 \pm \Delta\right)$$

In a similar manner, each shunt arm is:

$$(5)\quad \frac{Z_1}{j \sin\left[\frac{2\pi}{\lambda} l\right]} = \frac{Z_1}{j \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{4} \pm \frac{\lambda}{N}\right)\right]} = \frac{Z_1}{j \sin\left[\frac{\pi}{2} \pm \frac{2\pi}{N}\right]}$$

$$= \frac{-j Z_1}{\sin\frac{\pi}{2}\cos\frac{2\pi}{N}} = \frac{-j Z_1}{\cos\Delta} \doteq \frac{-j Z_1}{1 - \frac{\Delta^2}{2}} \doteq$$

$$-j Z_1 \left(1 + \frac{\Delta^2}{2}\right) \doteq -j Z_1$$

Therefore, to a first approximation the shunt arm is not affected by a small shift off design center. The elements X thus remain $-jZ_r$ and

$$(6)\quad S + 2X = Z_0 + 2jZ_r \pm 2j\Delta Z_r - 2jZ_r = Z_0 \pm 2j\Delta Z_r$$

Therefore, the last two determinants, 2 and 3, can be used by merely considering $Z_0 + 2\Delta Z_r$ as a special value of $Z_0$.

In 2 the driving voltages are added, opposite their associated meshes, in an external column at the right. This gives the current solutions. In the case being considered the number of driving voltages is usually one, never more than two; so the column will be zeros, except for one or two meshes. The current in any mesh is a fraction having D as denominator, and as numerator

the minor formed from D by substituting the e.m.f. column in the column corresponding to the mesh where the current is desired, i.e. column n if $I_n$ is desired.

Since D is common to all mesh current expressions, questions of relative power division between branches or of null balance can be handled by operations performed entirely with the numerator minors.

Some slight advantage in evaluating the numerator minors is gained by proceeding where possible so as to make $I_1$ or $I_n$ the desired current.

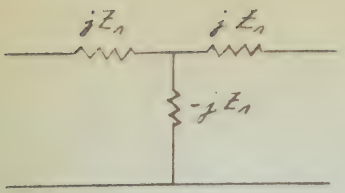
Alternatively, meshes where $Z_0 = 0$ can be chosen. Another needed quantity is driving point impedance. Since $I_n = \frac{Ed_n}{D}$, then $\frac{1}{Z_{DP}} = \frac{dn}{D}$ and $Z_{DP} = \frac{D}{dn}$. The resulting impedance will include an extra $Z_0$, the generator impedance, to which we must match.

Another method of analysis is by using equivalents. The equivalent T for a recurrent structure of constants Z (characteristic impedance) and P propagation constant (where $P = A + jB$) per unit length is as shown in Figure 18.

Once the circuit is diagrammed using the above equivalences, it can be reduced to simpler form by successive combinations of T's, by well known formulae.

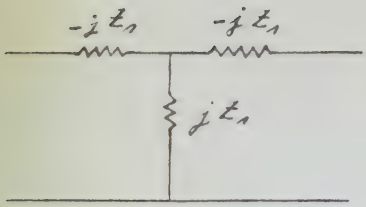
Now, we shall consider two special cases.

Case I. 1 1/2 $\lambda$ Ring with three arms used as a power divider two way.

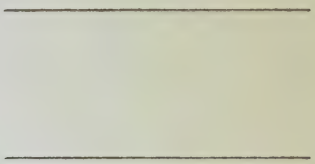



LENGTH = $\frac{\lambda}{4}$

LENGTH = $\frac{\lambda}{2}$
(T BECOMES INDETERMINATE)




LENGTH = $\frac{3}{4}\lambda$



LENGTH = $\lambda$

FIG. 19

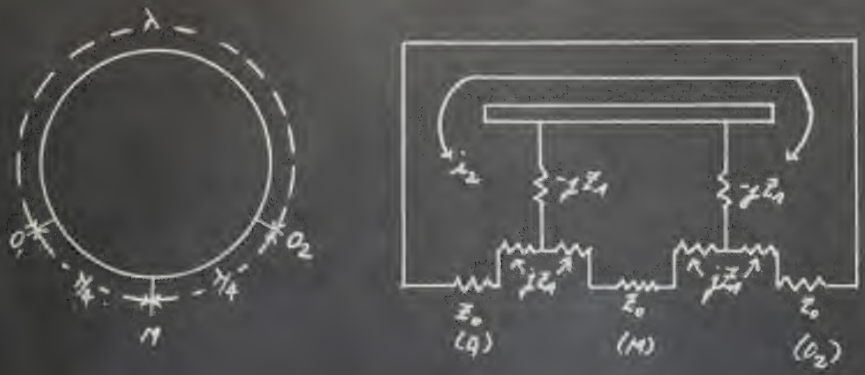



FIG 20

FIG 21

The method of equivalences will be used. The equivalent circuit is shown in Figure 20. From the figure it is seen that:

a. If power is fed in at E, it will divide equally between $O_1$ and $O_2$.

b. $O_1$ and $O_2$ are in proper wave length relationship for isolation relative to each other, but they are effectively in series and there will be no cancellation. Thus, the particular wavelength spacing is a necessary but not a sufficient condition.

c. If there is a voltage E at E, then we have

(7) $E = I_1 Z_o \qquad -I_2(2jZ_r)$

(8) $o = -I_1(-2jZ_r) + I_2(2Z_o)$ or

(9) $$\begin{vmatrix} E & Z_o & +2jZ_r \\ 0 & +2jZ_r & 2Z_o \end{vmatrix}$$ so

(10) $$\frac{I_1}{E} = \frac{2Z_o}{2Z_o^2 + 4Z_r^2}$$

The mesh impedance is then

(11) $$\frac{Z_o^2 + 2Z_r^2}{Z_o} = Z_o + \frac{2Z_r^2}{Z_o}$$

Therefore, an impedance match is secured if $2Z_r = Z_o$

d. For a voltage e at $O_1$, the current at $O_2$ is obtained from

(12) $$\begin{vmatrix} e & 2Z_o & -2jZ_r \\ o & -2jZ_r & Z_o \end{vmatrix}$$

$$\begin{vmatrix} & \\ & \end{vmatrix} \quad (23)$$

Thus

$$(13)\quad I_2 = \frac{eZ_o}{2Z_o^2 + 4Z_r^2}$$

For the impedance match condition of $\sqrt{2}Z_r = Z_o$, $I_2 = \frac{e}{4Z_o}$ which is one half the current which could be drawn through a load $Z_o$ connected to a source $Z_o$ with internal voltage e. Thus, there is a 6 db loss from $O_1$ to $O_2$.

Case II. 1 1/2 $\lambda$ ring with four arms used as a power dividers two way.

The method of determinants will be used. See Figure 21. Let $O_1$ be in Mesh 1. Then $D$ is in Mesh 4, $O_2$ is in Mesh 5 and M in Mesh 6. For meshes 2 and 3 $Z_o = 0$. Then the determinant 3 and its minor for mesh 5 with voltage applied at $O_1$, are respectively

$$D'_6 = \begin{vmatrix} Z_o & +jZ_r & 0 & 0 & 0 & jZ_r \\ jZ_r & 0 & jZ_r & 0 & 0 & 0 \\ 0 & jZ_r & 0 & jZ_r & 0 & 0 \\ 0 & 0 & jZ_r & Z_o & jZ_r & 0 \\ 0 & 0 & 0 & jZ_r & Z_o & jZ_r \\ jZ_r & 0 & 0 & 0 & jZ_r & Z_o \end{vmatrix}$$

and

$$d'_5 = \begin{vmatrix} jZ_r & 0 & jZ_r & 0 & 0 \\ 0 & jZ_r & 0 & jZ_r & 0 \\ 0 & 0 & jZ_r & Z_o & 0 \\ 0 & 0 & 0 & jZ_r & jZ_r \\ jZ_r & 0 & 0 & jZ_r & Z_o \end{vmatrix} = 0$$

$$\begin{vmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{vmatrix} =$$

$$\begin{vmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{vmatrix} =$$

Therefore, by adding outlet D, we have isolated branch $O_1$ from branch $O_2$. Since for this structure D is isolated from input at H, it must follow that an input at H will still divide equally between $O_1$ and $O_2$.

The band width over which a ring operates differentially is inherently narrower than that of a Tee; however, no matching stubs are required to achieve a fairly good standing wave ratio. The rings have considerably higher power capacity.

## APPENDIX C

### Analysis of the Direction Coupler as a Microwave Bridge

A directional coupler may be represented as an eight-terminal network similar to that shown in Figure 5 with the Tee replaced by the coupler. Let 2 and 4 be the secondary line and 1 and 3 the primary line. The amplitude transfer coefficient between the two lines is defined as

$$1.\quad K = \frac{E_{02}}{E_{13}} = \frac{E_{04}}{E_{11}}$$

when all lines are coupled to matched generators or loads. $E_{13}$ is the voltage amplitude of the wave traveling in toward the coupler in arm 3, $E_{02}$ is the voltage amplitude of the wave traveling out from the coupler in arm 2, etc. The directive property of the coupler is

$$2.\quad d = \frac{E_{04}}{E_{02}}$$

when the generator is coupled to 3. The reflection coefficient of the slots is $T_0$; the matched termination or reference match, $T_m$; and the test piece or unknown impedance, $T_t$. If the generator and detector are matched to the line,

$$3.\quad E_{02} = K(E_{13} + dE_{11})$$

also $$4.\quad E_{12} = T_m E_{02}$$

Substitute 4 in 3

$$5.\quad E_{12} = T_m K(E_{13} + dE_{11})$$

This wave is incident in the direction of the detector, but is reduced by coupling to the primary line and reflection by the slots. Ignoring small secondary reflections and couplings, we have

$$6.\quad E_{04} = BE_{12} + KE_{11} + KdE_{13}$$

where $$7.\quad B = 1 - |K|^2 - |T_c|^2$$

for conservation of energy. Also

$$8.\quad E_{01} = BE_{13} + T_c E_{11}$$

and 9 $$9.\quad E_{11} = T_t E_{01}$$

Substitute 9 in 8

$$10.\quad E_{11} = \frac{T_t BE_{13}}{1 - T_c T_t}$$

From 6

$$11.\quad E_{04} = BE_{13}K + \frac{BE_{13}KT_t(1-K)}{1 - T_c T_t} + \frac{KT_t BE_{13}}{1 - T_c T_t} + KdE_{13}$$

The second term is small compared to the others and $E_{04}$ may be approximated by

$$12.\quad E_{04} = KE_{13}\left(T_t B + \frac{T_t B}{1 - T_c T_t} + d\right)$$

Then the maximum and minimum values of the voltage to the detector are given by

$$13.\quad |E_{04}| = |K| \cdot |E_{13}| \left(\frac{|B| \cdot |T_t|}{1 - |T_c| \cdot |T_t|} \pm |B| \cdot |T_t| \pm |d|\right)$$

Since $|B|$ is very nearly unity, the phase error and imperfect reference match error enter in the same way as in the case of the

Tee. The factor d is the term in 13 which makes the directional coupler undesirable as a microwave bridge.

The directivity of a good directional coupler may be 30 to 35 db over a narrow band. Over a wide band, however, the directivity will be as low as 15 db for part of the band. This corresponds to /d/ >./ and the maximum error in measuring the voltage standing wave ratio of the test piece can be a factor of 1.22. A property of the directional coupler is that this error resulting from direct-coupling between the generator and the detector is frequency sensitive. In a Tee, an analagous error enters if the Tee is asymmetrical and there is cross-coupling between the series and parallel arms. This effect results only from asymmetry and is not frequency sensitive. Tee's may be constructed commercially so that the error from asymmetry does not exceed a factor of 1.01. This corresponds to a directivity of 52 db for a directional coupler.

## BIBLIOGRAPHY


1. ANALYSIS AND [illegible] OF WAVE GUIDE CIRCULAR RINGS FOR MICROWAVES; S. T. Sudenbom; Bell Technical Journal, July, 1948.
2. THE THEORY AND EXPERIMENTAL BEHAVIOR OF RIGHT ANGLE JUNCTIONS IN RECTANGULAR WAVE GUIDE, I. E. E. Journal, Sept. 1946.
3. HYBRID CIRCUITS FOR MICROWAVES, W. A. Tyrell, Proc. I. R. E. Nov. 1947.
4. [illegible] OF MICROWAVE MEASUREMENTS, C. G. Montgomery, Radiation Laboratory Series #11, McGraw-Hill.
5. A MICROWAVE DIPLEXER CIRCUIT, E. Chodorow, E. L. Ginzton, and F. Kane, Proc. I.R.E., June 1949.
6. [illegible] FOR MEASURING ATTENUATION AT MICROWAVE FREQUENCIES, L. E. Norton, Proc. I.R.E., July, 1949.
7. MICROWAVE PULSE PHASE MEASUREMENTS FOR WAVELENGTHS NEAR 12 and 32 MILLS, A. F. Kerzaiber, Proc, I. R. E., July 1949.
8. MICROWAVE MEASUREMENTS AND THEIR APPLICATIONS, F. J. Gaffney, Proc. I. R. E., October 1946.
9. MICROWAVE CONVERTERS, C. F. Edwards, Proc. I.R.E., Nov. 1947.
10. THE USE OF CIRCLES, BOTH JUNCTIONS AND CIRCLES, AS APPLIED TO [illegible] AND MICROWAVE CIRCUITS, E. O. Schneider, Proc, I.R.E. Aug. 1948.
11. A METHOD OF MEASURING PHASE AT MICROWAVE FREQUENCIES, S. E. Robertson, Bell System Technical Journal, Ja. 1949.

12. [illegible], S. A. Tyrell, Bell Telephone Laboratories Report [illegible]-42-160-10(ML122)

13. [illegible], Moreno, McGraw-Hill.

14. [illegible], Amullin and Montgomery, Radiation Lab. Series #14, McGraw-Hill.

15. [illegible], Ruyan, Radiation Lab. Series #9, McGraw-Hill.

16. [illegible], Raffney, Proc. I.R.E., Oct. 1946.

17. [illegible], R. H. Kyle, M.I.T.-643.

18. [illegible], Polytechnic Research and Development Company.

19. [illegible], G. Saxon and G.W. Miller, Wireless Engr., May 1948.

20. [illegible], W. A. Tynell, Bell Lab. Rec. Jan. 1948.

21. [illegible], K. S. Johnson, D. Van Nostrand Co. N.Y.

22. [illegible], S. A. Guillemin, Vols. I and II, John Wiley and Sons, Inc.

23. [illegible], W. H. Sharpless, Bell Tel. Lab. Report [illegible]-44-160-134.

24. [illegible], A. F. Pomeroy, Bell System Tech. Journal, July 1947.

25. [illegible], S. A. Schelkunoff, D. Van Nostrand Co.

26. [illegible], A. L. Samuel

and G. F. Crandell, Proc. I.R.E., Nov. 1948.

27. [illegible], V. K. Zworykin, Proc. I.R.E., February, 1947.

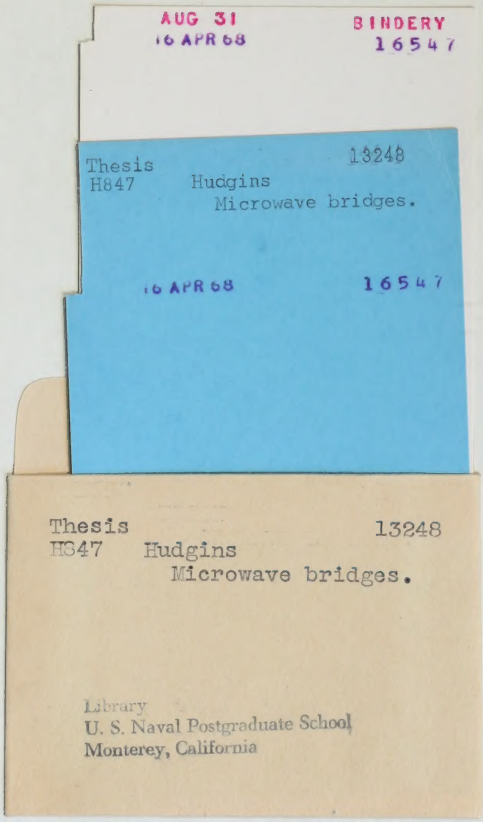
28. [illegible], J. C. Slater; M.I.T Technical Report No. 37, April 3, 1947.

VITA

Vance Robert Hudgins was born August 31, 1919, at Spartanburg, S. C. He attended elementary and secondary schools in South Carolina, completing high school at Spartanburg, S. C. in 1935. He attended Wofford College for one year and then entered the United States Naval Academy. Upon completion of the course of instruction at that institution in 1941, and upon receiving a B. S. degree, he entered the United States Marine Corps.

Mr. Hudgins has served continuously in the Marine Corps as a commissioned officer, both in the line and as a naval aviator. He has performed various command and staff duties including that of Squadron Commanding Officer, Group Personnel Officer, and instructor at the Marine Corps Schools. He has served three overseas tours during the last war and received several medals as commendation for excellent performance of duties. His present rank is Major U. S. M. C.

He entered the Post Graduate School at Annapolis, Md. in 1947 to take a graduate course in "Guided Missile Guidance." After one year at this institution, he transferred to The Johns Hopkins University to continue his graduate work for two more years in the Department of Electrical Engineering.




ARTISAN
GOLD LETTERING
AND
SMITH BINDERY
OAKLAND, CALIF.

thesH847
Microwave bridges.
3 2768 001 03547 0
DUDLEY KNOX LIBRARY